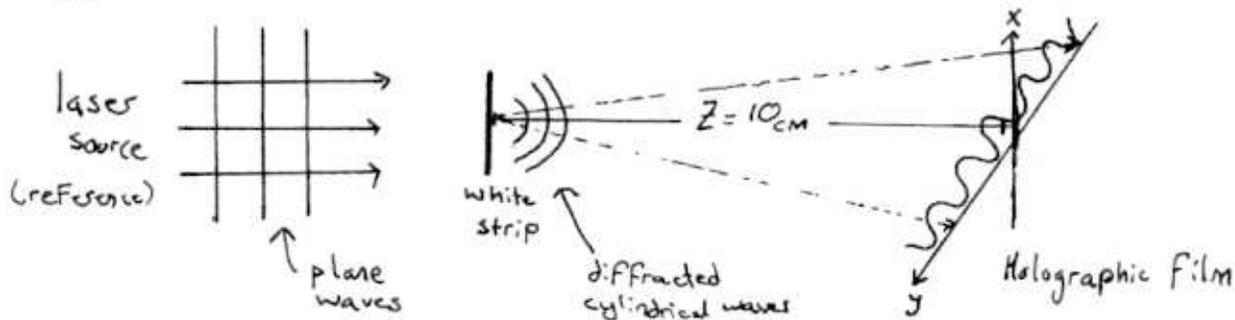


Physics 110B Homework #12

#1.



- The strip is in the x-direction, therefore the reference beam will only be able to be diffracted about the sides of the strip. The diffracted subject wave will be cylindrical and will form a diffracting grating pattern in the y-direction on superposition with the reference beam.

- On the film there will be maximums when

$$\sqrt{y^2 + z^2} - z = m\lambda \quad m=0, \pm 1, \pm 2, \dots$$

$$\Rightarrow y_{\text{max}} = \left((m\lambda + z)^2 - z^2 \right)^{1/2} \quad \Rightarrow m = \left(\sqrt{y_m^2 + z^2} - z \right) / \lambda$$

$$\Rightarrow y_{m+1} = \left(((m+1)\lambda + z)^2 - z^2 \right)^{1/2}$$

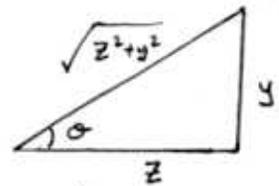
$$= \left(\left(\sqrt{y_m^2 + z^2} + \lambda \right)^2 - z^2 \right)^{1/2}$$

$$= \left(y_m^2 + z^2 + \lambda^2 + 2\lambda\sqrt{y_m^2 + z^2} - z^2 \right)^{1/2}$$

$$= \left(y_m^2 + \lambda^2 + 2\lambda\sqrt{y_m^2 + z^2} \right)^{1/2}$$

$$\Rightarrow y_{m+1} - y_m = \left(y_m^2 + \lambda^2 + 2\lambda\sqrt{y_m^2 + z^2} \right)^{1/2} - y_m = \delta$$

distance diffracted wave travels



distance plane wave travels

$$\lambda = 633 \times 10^{-9} \text{ m}$$

$$z = .1 \text{ m}$$

So,

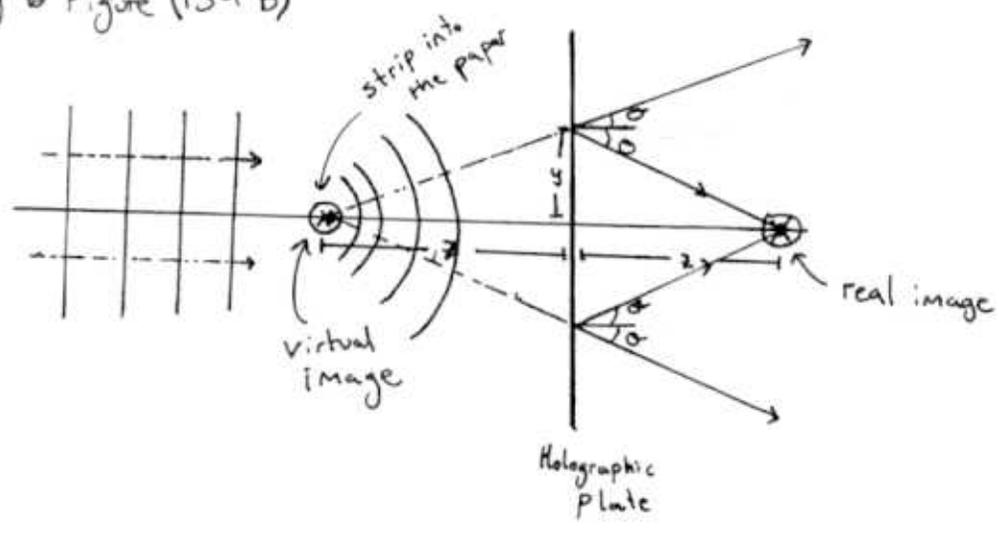
$$S = \left(y^2 + (633 \times 10^{-9})^2 + 2 \cdot 633 \times 10^{-9} \sqrt{y^2 + .1^2} \right)^{1/2} - y$$

For

$y = 0$	\Rightarrow	$S = 3.56 \times 10^{-4} \text{ m}$
$y = .01 \text{ m}$	\Rightarrow	$S = 6.36 \times 10^{-6} \text{ m}$
$y = .02 \text{ cm}$	\Rightarrow	$S = 3.23 \times 10^{-6} \text{ m}$
$y = .04 \text{ cm}$	\Rightarrow	$S = 1.70 \times 10^{-6} \text{ m}$

#2. The same argument Pedrotti gives on pages 267-269 for a point source will also be true for the thin strip.

In analogy to Figure (13-1 b)



- When the plane wave hits the holographic plate it will diffract. At an angle of θ there will form constructively a real image on the positive z-side of the plate. This image is formed by the reference beam diffracted inwards, symmetrically the beam will diffract outwards and form a virtual holographic image on the left side of the plate.

- In the case of a holographic plate, we have a semi-continuous zone pattern, therefore there are no second or higher order diffracted beams.

Now, from geometrical considerations the angle of diffraction for various heights y will be:

$$\Rightarrow \tan \theta = \frac{y}{z}$$

So,

$$y = 1 \text{ cm} \quad \theta = \tan^{-1} \frac{1 \text{ cm}}{10 \text{ cm}} = 5.71^\circ$$

$$y = 2 \text{ cm} \quad \theta = \tan^{-1} \frac{2 \text{ cm}}{10 \text{ cm}} = 11.3^\circ$$

$$y = 4 \text{ cm} \quad \theta = \tan^{-1} \frac{4 \text{ cm}}{10 \text{ cm}} = 21.8^\circ$$

#3 (Pedrotti 13-1)

From eq. (10.13)

$$I = I_1 + I_2 + 2\sqrt{I_1 I_2} \cos \delta,$$

$$\delta = (\vec{k}_1 - \vec{k}_2) \cdot \vec{r} + (\epsilon_1 - \epsilon_2)$$

There is no delay between interaction of incident beam with object + production of scattered wave:

$$\epsilon_1 - \epsilon_2 = 0$$

Also, $(\vec{k}_1 - \vec{k}_2) \cdot \vec{r} = kr \cos \theta - kr$

From Pythagorem's Theorem.

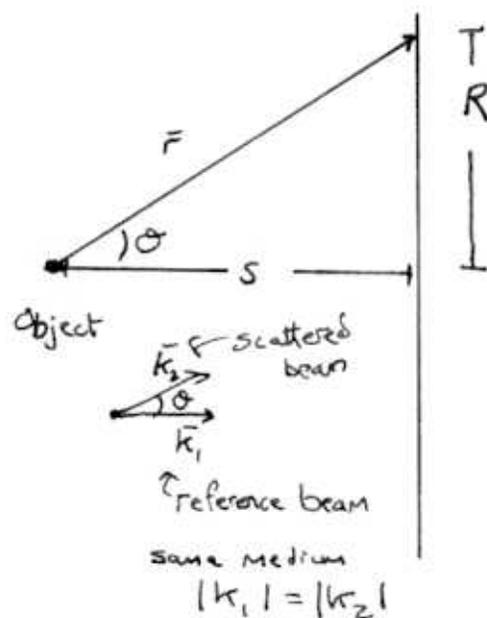
$$r = \sqrt{s^2 + R^2} = s \sqrt{1 + R^2/s^2} \approx s + R^2/2s$$

And, $\cos \theta = \frac{s}{\sqrt{s^2 + R^2}} \approx 1 - \frac{R^2}{2s^2}$

$$\begin{aligned} \Rightarrow \delta &= k \left(s + \frac{R^2}{2s} \right) \left(1 - \frac{R^2}{2s^2} \right) - k \left(s + \frac{R^2}{2s} \right) \\ &= -\frac{kR^2}{2s} + \mathcal{O}\left(\frac{R^4}{s^3}\right) \end{aligned}$$

$$\Rightarrow \cos \delta \approx \cos 2aR^2; \quad a = \frac{\pi}{2s\lambda} = \frac{k}{4s}$$

$$\begin{aligned} \Rightarrow I &= I_1 + I_2 + 2\sqrt{I_1 I_2} \cos 2aR^2 = I_1 + I_2 + 2\sqrt{I_1 I_2} (\cos^2 aR^2 - \sin^2 aR^2) \\ &= I_1 + I_2 + 2\sqrt{I_1 I_2} (2\cos^2 aR^2 - 1) = \boxed{I_1 + I_2 - 2\sqrt{I_1 I_2} + 4\sqrt{I_1 I_2} \cos^2 aR^2} \end{aligned}$$



#4 (Pedrotti 13-12)

(a) From the previous problem we have

$$I_F = A + B \cos^2 a R^2$$

which can be rewritten as,

$$= A + B \left(\frac{1 + \cos 2a R^2}{2} \right) = \left(A + \frac{B}{2} \right) + \frac{B}{4} e^{2iaR^2} + \frac{B}{4} e^{-2iaR^2}$$

We now write down the equation for the plane wave reference beam.
The object will be the origin $r=0$, thus

$$E_R = E_0 e^{i(\omega t - ks)} \quad \leftarrow \text{ } s \text{ here because this gives the amplitude at the screen } r=s, \text{ while } r=0 \text{ is at the object}$$

From eq (13-9)

$$E_H \propto I_F E_R = \left(\underbrace{\left(A + \frac{B}{2} \right)}_{\textcircled{1}} + \underbrace{\frac{B}{4} e^{i\kappa R^2/2s}}_{\textcircled{2}} + \underbrace{\frac{B}{4} e^{-i\kappa R^2/2s}}_{\textcircled{3}} \right) E_0 e^{i(\omega t - ks)}$$

The subject beam has the form of a spherical wave,

$$\begin{aligned} E_S &= \frac{D}{r} e^{i(\omega t - \kappa r)} \approx \frac{D}{s} \left(1 - \frac{R^2}{2s^2} \right) e^{i(\omega t - \kappa s - \kappa R^2/2s)} \\ &= D' e^{i(\omega t - \kappa s - \kappa R^2/2s)} \end{aligned}$$

Therefore, the first term of the boxed equation corresponds to

$$\textcircled{1} \quad E_{H1} = \left(A + \frac{B}{2} \right) e^{i(\omega t - \kappa s)} \quad \bullet \text{ the reference beam modulated in amplitude but not in phase}$$

The second term ② corresponds to

$$\textcircled{2} \quad E_{H2} = \frac{B}{4} e^{i(\omega t - ks + kR^2/2s)}$$

- the subject beam modulated in amplitude and has a phase reversal (+ sign in front of $kR^2/2s$)
- Produces phase reversed real image

The third term ③ corresponds to

$$\textcircled{3} \quad E_{H3} = \frac{B}{4} e^{i(\omega t - ks - kR^2/2s)}$$

- the virtual image with correct phase

(b) From the previous problem we have

$$r \approx s + \frac{R^2}{2s}$$

$$\Rightarrow \delta = k(r-s) \approx \frac{2\pi}{\lambda} \left(\frac{R^2}{2s} \right) = \frac{\pi R^2}{\lambda s} = \delta$$

For the subject beam we have

$$E_s = D' e^{i(\omega t - ks - \underbrace{kR^2/2s}_{\delta})}$$

If the sign of the phase is reversed we have,

$$E_s = D' e^{i(\omega t - ks + kR^2/2s)}$$

which is just E_{H2} which is a converging real image

#5 (Pedrotti 22-1)

(a) From eq. (22-6)

$$r = R \left(1 + \frac{x^2 + y^2}{R^2} \right)^{1/2}$$

$$\boxed{r \approx R + \frac{x^2 + y^2}{2R}} \Leftrightarrow \frac{x^2 + y^2}{R^2} \ll 1$$

Thus, from eq. (22-5)

$$(b) \Rightarrow \boxed{\tilde{E}(x, y, z=R) \approx C e^{ikR} e^{ik(x^2 + y^2)/2R}}$$

#6 (Pedrotti 22-11)

$$\frac{\Phi(r=a)}{\Phi_{tot}} = \frac{1}{\Phi_{tot}} \iint_{\text{aperture}} |\tilde{E}(x, y, z, t)|^2 dA$$

using eq. (22-18)

$$= \frac{1}{\Phi_{tot}} \iint_A |E_0 e^{ik(x^2 + y^2)/2R(z)} e^{-(x^2 + y^2)/w^2(z)} e^{i(kz + p(z) - \omega t)}|^2 dA$$

$$= \frac{E_0^2}{\Phi_{tot}} \iint_A e^{-2(x^2 + y^2)/w^2(z)} dA$$

$$= \frac{E_0^2}{\Phi_{tot}} \int_0^a r dr \int_0^{2\pi} d\phi e^{-2r^2/w^2(z)}$$

$$= E_0^2 \cdot \frac{2}{E_0^2 \pi w^2(z)} \cdot \overset{\Phi_{tot} \text{ (from page 473)}}{2\pi} \cdot \frac{w^2(z)}{4} \cdot (1 - e^{-2a^2/w^2(z)})$$

$$\Rightarrow \boxed{\frac{\Phi(r=a)}{\Phi_{tot}} = 1 - e^{-2a^2/w^2(z)}}$$

#7 (Pedrotti 22-15)

From eq. (22-58)

$$H_m(\xi) = (-1)^m e^{\xi^2} \frac{d^m}{d\xi^m} (e^{-\xi^2})$$

$$m=0 \Rightarrow H_0(\xi) = e^{\xi^2} \cdot e^{-\xi^2} = 1 \checkmark$$

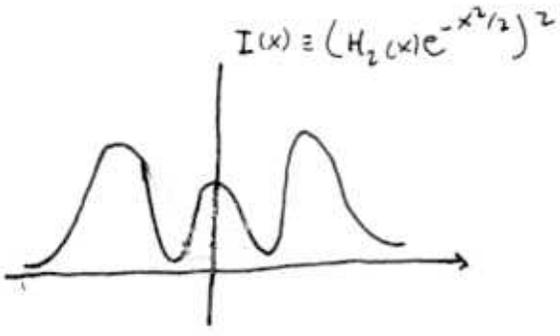
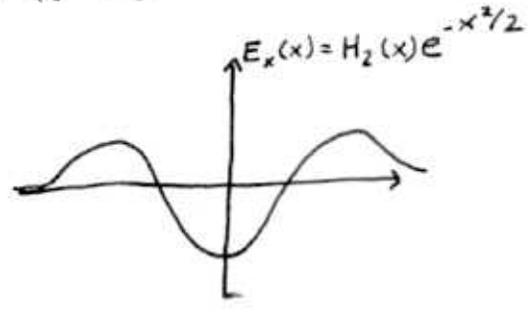
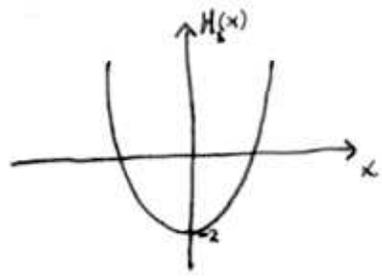
$$m=1 \Rightarrow H_1(\xi) = (-1) e^{\xi^2} \frac{d}{d\xi} e^{-\xi^2} = 2\xi \checkmark$$

$$m=2 \Rightarrow H_2(\xi) = e^{\xi^2} \frac{d^2}{d\xi^2} (e^{-\xi^2}) = e^{\xi^2} (4\xi^2 e^{-\xi^2} - 2e^{-\xi^2}) = 4\xi^2 - 2 \checkmark$$

#8 (Pedrotti 22-17)

Following Figure 22-17

$$H_{m=2}(x) = \frac{8x^2}{w^2} - 2 \Rightarrow E \propto \left(\frac{8x^2}{w^2} - 2\right) e^{-(x^2+y^2)/2}$$



Burn Pattern:

